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BOND SHEAR STRENGTH FOR ADHESIVE BONDED DOUBLE-LAP JOINTS

LIYONG TONG
Cooperative Research Center for Aerospace Structures, 361 Milperra Road, P.O. Box 30, Bankstown, NSW, Australia 2200

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Abstract—For adhesively bonded double-lap joints with an arbitrarily nonlinear shear stress–strain property, a simple formula is developed for predicting the bond shear strength developable between the adherends for the cohesive shear failure mode. In this formula, the bond shear strength is characterized by the maximum shear strain energy density of the adhesive. The thermal mismatch effect is highlighted in terms of reducing the bond shear strength for balanced joints, and in terms of either decreasing or increasing the bond shear strength for the stiffness imbalanced joints.

1. INTRODUCTION

Adhesive bonded joints are increasingly being used in metal and composite structures in aerospace and automotive industries. Unlike bolted, riveted or other types of mechanically fastened joints, the loads acting on the adhesive bonded joints are transferred through the adhesive mainly in shear in certain circumstances. Thus, prediction of the bond shear strength, especially with a simple and efficient formula, plays an important role at the preliminary design stage. There are various types of adhesive bonded joints, e.g. single-lap joints, double-lap joints, single-strap joints, double-strap joints, scarf joints, stepped joints and tapered joints, etc. In this study, our attention will be focused on the prediction of the bond shear strength for adhesive bonded double-lap joints. Various theoretical analyses have been given by many previous researchers. The pioneer works by Volkersen (1938) for double-lap joints and by Goland and Reissner (1944) for single-lap joints provided a basic understanding of the qualitative behavior of the joints under tensile loads. Hart-Smith (1973) studied adhesive bonded double-lap joints using elastic–plastic analytical techniques. Explicit solutions obtained include sufficiently simple formulas for predicting the shear bond strength and the plastic zone length. It is shown that for a given double-lap joint with a particular geometric configuration and specified material properties including ideal elastic–plastic or bi-elastic adhesives, the maximum bond shear strength developable between specified adherends can be characterized by the adhesive strain energy in shear per unit bonded area. Hart-Smith's formula is limited to the elastic–plastic and bi-elastic adhesive models. Similarly, ESDU (1979) pointed out that the precise shape of the shear stress–strain curve has no effect on the limiting joint strength, and can affect only the adhesive shear stress distribution along the overlap. Once again, a particular adhesive model is utilized in ESDU's analysis.

The simple formula of the bond shear strength provides an efficient tool for designing bonded joints [see e.g. Thrall (1979), Jones *et al.* (1993)], and bonded repairs [see e.g. Hart-Smith (1988), Baker and Jones (1988)], etc.

In this study an arbitrary shear stress–strain model is used as a basis for developing the formula of the bond shear strength. In this case, closed-form solutions for the shear strain and stress distribution are mathematically intractable. However, the bond shear strength can be determined without completely solving the governing equation. It is shown that bond shear strengths can be characterized by the maximum strain energy density in shear for the adhesive. The formula of the bond shear strength is also presented to include the thermal mismatch effect between the thermal expansion coefficients of the two adherends.

2. PROBLEM FORMULATION

Consider a symmetric adhesive bonded double-lap joint, as shown in Fig. 1, where l is the length of the overlap and x is measured from the middle point of the overlap. The inner adherend is assumed to be subjected to tensile load P at $x = 0.5l$, and the outer adherends are loaded with $0.5P$ at $x = -0.5l$. To determine the ultimate load or the bond shear strength P, the shear lag model is used by neglecting the transverse deflection. In the shear lag model the longitudinal displacements for the adherends are assumed to be constant across the adherend thickness. Using the symmetric condition, the longitudinal equilibrium equations for the outer and inner adherends can be written as

$$\frac{\mathrm{d}T_o}{\mathrm{d}x}+\tau = 0, \quad \frac{\mathrm{d}T_i}{\mathrm{d}x}-2\tau = 0, \tag{1}$$

where T_o and T_i denote the longitudinal forces acting on the outer and inner adherends. τ is the shear stress in the adhesive acting on both adherends, and is assumed to be constant across the bondline thickness.

The longitudinal forces can be expressed in terms of the longitudinal displacements u_o and u_i as follows

$$T_o = E_o t_o \frac{\mathrm{d}u_o}{\mathrm{d}x}, \quad T_i = E_i t_i \frac{\mathrm{d}u_i}{\mathrm{d}x}, \tag{2}$$

where E_o and E_i are the Young's moduli of the outer and inner adherends; t_o and t_i denote the thickness of the outer and inner adherends; u_o and u_i are the longitudinal displacements of the outer and inner adherends.

As an approximation, the adhesive shear strain is assumed to be constant through the thickness (Carpenter, 1991) and is given by

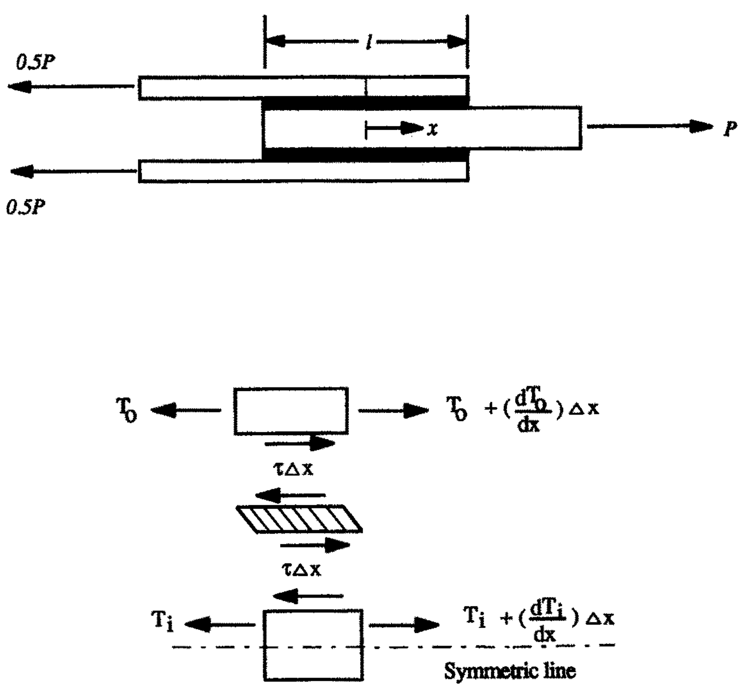

Fig. 1. Geometry and notation for adhesive bonded double-lap joint.

$$\gamma = \frac{u_i - u_o}{t} \tag{3}$$

where t is the bondline thickness.

Substituting eqns (2) into eqns (1) yields

$$E_o t_o \frac{d^2 u_0}{dx^2} + \tau = 0 \tag{4}$$

$$E_i t_i \frac{d^2 u_i}{dx^2} - 2\tau = 0, \tag{5}$$

and using eqns (3), (4) and (5) can be rewritten in terms of the shear strain γ as follows:

$$\frac{d^2\gamma}{dx^2} - \lambda^2 \tau = 0 \tag{6}$$

where

$$\lambda^2 = \frac{1}{t}\left(\frac{1}{E_o t_o} + \frac{2}{E_i t_i}\right). \tag{7}$$

Equation (6) is a second-order differential equation of the shear strain γ. It is mathematically tractable only when the shear stress of the adhesive τ is a simple function of the shear strain γ, for example, when the shear stress is a linear function of the shear strain. However, eqn (6) does not permit a general closed-form solution when the shear stress–strain behavior of the adhesive is assumed to take the following form

$$\tau = \tau(\gamma), \tag{8}$$

where $\tau(\gamma)$ is an arbitrary function of γ, see Fig. 2(a), and could be the shear stress–strain curve of the adhesive measured with the thick adherend test specimen (Krieger, 1988). Experiments show that $\tau(\gamma)$ could be rate-dependent, temperature-dependent and *in situ* environment-dependent, etc (Jones *et al.*, 1993).

Substituting eqn (8) into (6) yields

$$\frac{d^2\gamma}{dx^2} - \lambda^2 \tau(\gamma) = 0, \tag{9}$$

which is a nonlinear second-order differential equation of the shear strain γ and does not permit a closed-form solution for γ. However, for some special cases of the shear stress–strain curves of the adhesives, closed-form solutions are tractable. Hart-Smith (1973) discussed the following two special types of shear stress–strain behaviors.

(a) Ideal elastic–plastic stress–strain behavior in shear for the adhesive, namely,

$$\tau = \begin{cases} G\gamma & \gamma < \gamma_e \\ \tau_p & \gamma_e \leqslant \gamma \leqslant \gamma_p \end{cases} \tag{10}$$

where γ_e, γ_p and G are chosen in such a way that the maximum strain energy density in shear computed using the ideal elastic–plastic model is identical to that computed using the measured shear stress–strain curve for the adhesive [see Fig. 2(a)].

(b) Bi-elastic stress–strain behavior in shear for the adhesive, namely,

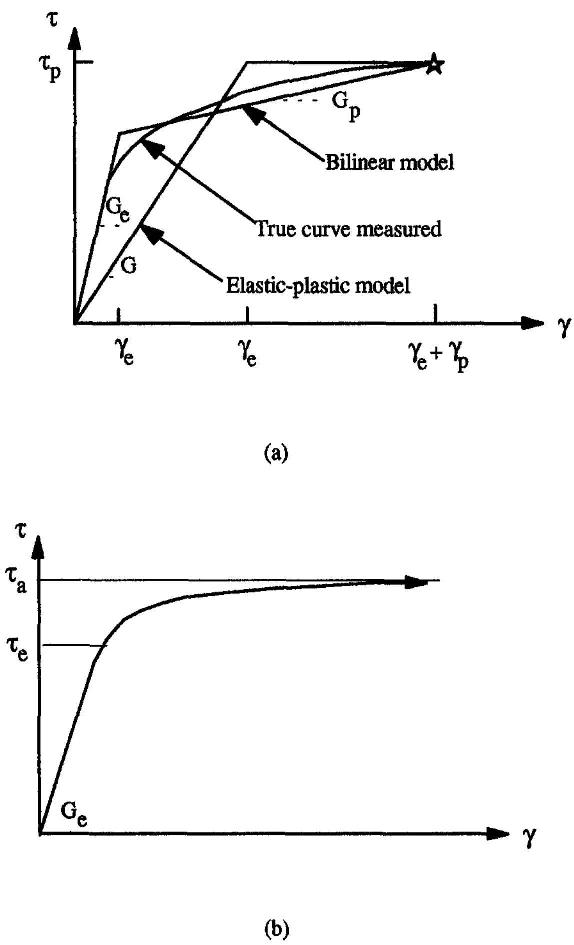

Fig. 2. Stress–strain properties in shear for adhesive.

$$\tau = \begin{cases} G_e\gamma & \gamma < \gamma_e \\ \tau_e + G_p(\gamma-\gamma_e) & \gamma_e \leqslant \gamma \leqslant \gamma_p \end{cases} \tag{11}$$

where γ_e, γ_p, G_e and G_p are chosen in the same way as that used for the elastic–plastic model [see Fig. 2(a)].

Another adhesive model used in analysing the inelastic shear stress and strain in the adhesive bonded lap joints loaded in tension or shear (ESDU, 1979) is shown in Fig. 2(b), and the following expressions are assumed to represent the shear stress–strain properties of the adhesive

$$\tau = \begin{cases} G_e\gamma & \gamma < \gamma_e \\ \tau_e + \dfrac{\alpha\beta}{\alpha+\beta} & \gamma_e \leqslant \gamma \end{cases} \tag{12}$$

where

$$\alpha = G_e\gamma - \tau_e, \quad \beta = \tau_a - \tau_e. \tag{13}$$

Equations (12) are chosen for convenience of calculation (ESDU, 1979).

The boundary conditions at both ends of the overlap, as shown in Fig. 1, are given by

$$E_o t_o \frac{du_o}{dx} = \frac{P}{2}, \quad E_i t_i \frac{du_i}{dx} = 0 \quad \text{when } x = -\frac{l}{2} \tag{14a}$$

$$E_o t_o \frac{du_o}{dx} = 0, \quad E_i t_i \frac{du_i}{dx} = \mathrm{P} \quad \text{when } x = \frac{l}{2}. \tag{14b}$$

Noting eqn (3), the above boundary conditions can be rewritten in terms of the shear strain γ as follows

$$\frac{d\gamma}{dx} = -\frac{P}{2E_o t_o t} \quad \text{when } x = -\frac{l}{2} \tag{15a}$$

$$\frac{d\gamma}{dx} = \frac{P}{E_i t_i t} \quad \text{when } x = \frac{l}{2}. \tag{15b}$$

Evidently a general closed-form solution for eqn (9) and the related boundary conditions (15) are intractable. However, a simple explicit expression for the ultimate load or bond shear strength P could be determined without obtaining the complete solution for eqns (9) and (15). This simple formula plays an important role in the preliminary design of the double-lap joints.

3. DETERMINATION OF BOND SHEAR STRENGTH P

To determine the bond shear strength P for the double-lap joints, we need to integrate eqn (9). For an arbitrary shear stress–strain property, it is difficult to obtain a mathematically tractable solution for γ by following the same procedure used in Hart-Smith (1973). However, we can integrate eqn (9) for determining the bond shear strength or the ultimate load P without attaining the detailed shear strain and stress distribution. By multiplying $2(d\gamma/dx)$ on both sides of eqn (9), we have

$$2\frac{d^2\gamma}{dx^2}\frac{d\gamma}{dx} - 2\lambda^2\tau(\gamma)\frac{d\gamma}{dx} = 0 \tag{16}$$

which can be rewritten as

$$\frac{d}{dx}\left(\frac{d\gamma}{dx}\right)^2 - 2\lambda^2\tau(\gamma)\frac{d\gamma}{dx} = 0 \tag{17}$$

or in the form of a complete differentiation

$$d\left(\frac{d\gamma}{dx}\right)^2 - 2\lambda^2\tau(\gamma)\,d\gamma = 0. \tag{18}$$

Evidently, eqn (18) is valid for any point in the overlap (namely, $-0.5l \leqslant x \leqslant 0.5l$). Integration of eqn (18) with respect to $d\gamma/dx$ for the first term and with respect to γ for the second term yields

$$\left(\frac{d\gamma}{dx}\right)^2\Bigg|_0^{d\gamma/dx} - 2\lambda^2\int_0^{\gamma}\tau(\gamma)\,d\gamma = 0 \tag{19}$$

or

$$\left(\frac{d\gamma}{dx}\right)^2 = 2\lambda^2 \int_0^{\gamma} \tau(\gamma)\,d\gamma. \tag{20}$$

It is worth pointing out that it is assumed that there is no initial shear strain or shear stress. Similar to eqn (18), eqn (20) holds for any point in the overlap of the joint, namely, $-0.5l \leqslant x \leqslant 0.5l$. Equation (20) can be physically interpreted as: the slope of the shear strain distribution at any point is related to the strain energy density in shear computed using the shear strain at that point. Noting eqn (2), eqn (20) can be written as

$$\left(\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o}\right)^2 = 2\lambda^2 \int_0^{\gamma} \tau(\gamma)\,d\gamma. \tag{21}$$

Equation (21) reveals that the difference between the average longitudinal stresses acting on the inner and outer adherends at any point is related to the shear strain density computed using the shear strain at that point. Although eqn (21) cannot give us a closed-form solution for γ, it can be used to determine the ultimate load P. As shown in Fig. 1, $T_o = 0$ and $T_i = P$ at $x = 0.5l$; $T_o = 0.5P$ and $T_i = 0$ at $x = -0.5l$. When the load P at one end of the overlap ($x = 0.5l$) is transferred through the adhesive to the other end of the overlap ($x = -0.5l$), T_o increases from 0 to $0.5P$ and T_i decreases from P to 0. Evidently γ and $d\gamma/dx$ attain their maximum value at either the left end ($x = -0.5l$) or the right end ($x = 0.5l$) of the overlap, namely,

$$\left(\frac{P}{2E_o t_o l}\right)^2 = 2\lambda^2 \int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma \quad \text{when the joint fails at } x = -0.5l \tag{22}$$

or

$$\left(\frac{P}{E_i t_i l}\right)^2 = 2\lambda^2 \int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma \quad \text{when the joint fails at } x = 0.5l. \tag{23}$$

Equations (22) and (23) can be rearranged in the following form

$$P = 2E_o t_o l \sqrt{2\lambda^2 \int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma} \quad \text{when the joint fails at } x = -0.5l \tag{24}$$

or

$$P = E_i t_i l \sqrt{2\lambda^2 \int_0^{\gamma_{max}} \tau(\gamma)\,d\gamma} \quad \text{when the joint fails at } x = 0.5l. \tag{25}$$

The maximum load P_{max} is the lesser one of the two values for P computed using eqns (24) and (25).

Case 1

For unbalanced double-lap joints (i.e. $2E_o t_o \neq E_i t_i$): the shear strain may attain its maximum value at either one end or the other of the overlap.

(a) When $2E_o t_o \geqslant E_i t_i$, the joint fails at $x = 0.5l$ and the bond shear strength takes the form

$$P_{\max} = E_i t_i t \sqrt{2\lambda^2 \int_0^{\gamma_{\max}} \tau(\gamma)\,d\gamma}. \tag{26}$$

(b) When $2E_o t_o \leqslant E_i t_i$, the joint fails at $x = -0.5l$ and the bond shear strength is given by

$$P_{\max} = 2E_o t_o t \sqrt{2\lambda^2 \int_0^{\gamma_{\max}} \tau(\gamma)\,d\gamma}. \tag{27}$$

Case 2

For balanced double-lap joints (i.e. $2E_o t_o = E_i t_i$): The joint fails simultaneously at both ends of the overlap and the maximum load takes the following form

$$P_{\max} = 4\sqrt{E_o t_o t \int_0^{\gamma_{\max}} \tau(\gamma)\,d\gamma}. \tag{28}$$

Equations (26)–(28) reveal that the maximum load P can be characterized in terms of the maximum strain energy density in shear obtainable in the adhesive. The maximum strain energy density in shear can be computed using the shear stress–strain data measured with the thick adherend test. The effect of stiffness imbalance on the bond shear strength can be investigated through introducing the following notation

$$k_{\text{stiffness}} = \frac{P_{\max}}{4\sqrt{E_o t_o t \int_0^{\gamma_{\max}} \tau(\gamma)\,d\gamma}}. \tag{29}$$

Noting eqns (7), (26) and (27), eqn (29) can be expressed in terms of the relative stiffness of the adherends as follows:

$$k_{\text{stiffness}} = \begin{cases} \dfrac{1}{\sqrt{2}}\sqrt{1+\left(\dfrac{2E_o t_o}{E_i t_i}\right)} & \text{when } \dfrac{2E_o t_o}{E_i t_i} \leqslant 1.0 \\ \dfrac{1}{\sqrt{2}}\left(\dfrac{E_i t_i}{2E_o t_o}\right)\sqrt{1+\left(\dfrac{2E_o t_o}{E_i t_i}\right)} & \text{when } \dfrac{2E_o t_o}{E_i t_i} > 1.0. \end{cases} \tag{30}$$

Equations (29) and (30) indicate the effect of changing the inner adherend stiffness when prescribing the stiffness of the outer adherends. The effect of the stiffness imbalance on the nondimensional bond shear strength is plotted in Fig. 3. Apparently for the stiffness balanced joints (namely $2E_o t_o = E_i t_i$), shear failure happens at both ends of the overlap and $k_{\text{stiffness}}$ attains its maximum value (i.e. $k_{\text{stiffness}} = 1.0$). Any imbalance in stiffness tends to cause shear failure at one end of the overlap and thus decreases the bond shear strength.

It is worth noting that Hart-Smith (1973) developed the full solutions for the shear stress in the joints using both ideal elastic–plastic and bi-elastic behaviors of the adhesive. The formulas for bond shear strength were finally given as follows:

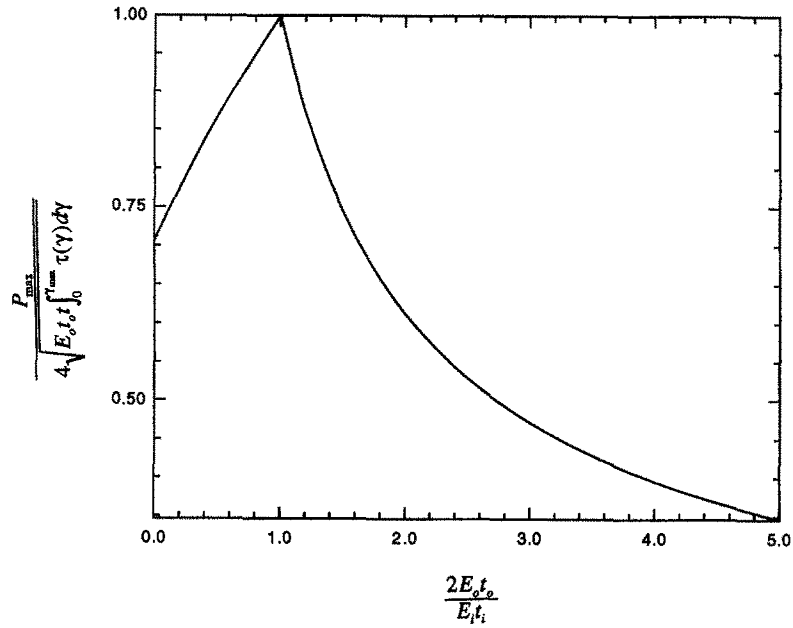

Fig. 3. Effect of stiffness imbalance on nondimensional bond shear strength with prescribed outer adherend stiffness (i.e. $2E_o t_o$ = constant).

for ideal elastic–plastic adhesive:

$$P_{\max} = 4\sqrt{E_o t_o t \tau_p \left(\frac{\gamma_e}{2} + \gamma_p\right)}, \tag{31}$$

and for bi-elastic adhesive:

$$P_{\max} = 4\sqrt{E_o t_o t [\tfrac{1}{2}(G_e \gamma_e^2 + G_p \gamma_p^2) + G_e \gamma_e \gamma_p]}. \tag{32}$$

As shown in Fig. 2(a), the terms $\tau_p(0.5\gamma_e + \gamma_p)$ and $0.5(G_e\gamma_e^2 + G_p\gamma_p^2) + G_e\gamma_e\gamma_p$ in eqns (31) and (32) represent the area of the stress–strain curves for ideal elastic–plastic and bi-elastic adhesive models, and are identical to the term $\int_0^{\gamma_{\max}} \tau(\gamma)\,d\gamma$ in eqn (28). Thus it is evident that the bond shear strength formula in eqn (28) for the joints with an arbitrary adhesive shear property is an extension of Hart-Smith's formula in eqns (31) or (32).

4. THERMAL MISMATCH

Metal and composite adherends need to be frequently bonded together to reduce the stress concentration associated with bolt holes etc. Due to the difference in their thermal expansion coefficients, the normally high shear bond strength may be decreased, for example, in graphite–epoxy-to-aluminum bonding or repairing [see e.g. Hart-Smith (1988) and Baker and Jones (1988)]. In this section the thermal mismatch effect will be discussed.

To understand the thermal mismatch effect, let us assume that there is a difference ΔT between the initial reference temperature T_{iref} and the final reference temperature T_{fref}, thus the constitutive relations in eqns (2) need to be revised as:

$$T_o = E_o t_o \left(\frac{du_o}{dx} + \alpha_o \Delta T\right), \quad T_i = E_i t_i \left(\frac{du_i}{dx} + \alpha_i \Delta T\right) \tag{33}$$

where α_i and α_o are the coefficients of thermal expansion for the inner and outer adherends.

It is assumed that α_i is not equal to α_o. By including the temperature effect in the shear stress–strain behaviors, the governing equation in eqn (9) may be modified as

$$\frac{d^2\gamma}{dx^2}-\lambda^2\tau(\gamma, T)=0 \tag{34}$$

where T ranges from T_{iref} to T_{fref}, and is assumed to be constant across the overlap. The shear stress–strain behavior for the adhesive varies at different temperatures as shown in Fig. 4 (Hart-Smith, 1973).

Similar to the preceding section, integration of eqn (33) yields

$$\left(\frac{d\gamma}{dx}\right)^2\bigg|_{\text{final}}-\left(\frac{d\gamma}{dx}\right)^2\bigg|_{\text{initial}}=2\lambda^2\int_{\gamma_{\text{initial}}}^{\gamma_{\text{final}}}\tau(\gamma, T)\,d\gamma \tag{35}$$

where γ_{initial} and γ_{final} are the shear strain at the initial state and the final state; $d\gamma/dx|_{\text{initial}}$ and $d\gamma/dx|_{\text{final}}$ are the slope of the shear strain at the initial and the final state. Now let us discuss the following two cases.

Case 1

In this case, the temperature difference ΔT_{cr} is defined as the difference between the adhesive cure temperature T_{cure} and the room temperature T_{room} introduced when cooling down in the autoclave. It is assumed that the adhesive cure temperature is a strain-free temperature and there is no mechanical loads acting on the joint. Using eqn (3), eqn (35) can be written as

$$\left[\frac{(\alpha_i-\alpha_o)\Delta T_{cr}}{t}\right]^2=2\lambda^2\int_0^{\gamma_{\text{final}}}\tau(\gamma, T)\,d\gamma. \tag{36}$$

When γ_{final} attains its maximum value $\gamma_{\max}$, the allowable temperature difference $\Delta T_{cr}^{\text{allowable}}$ for breaking the joint apart when cooling down is given by

$$|\Delta T_{cr}^{\text{allowable}}|=\frac{\lambda t}{|\alpha_i-\alpha_o|}\sqrt{2\int_0^{\gamma_{\max}}\tau(\gamma, T)\,d\gamma}. \tag{37}$$

As an approximation in eqn (37), the room temperature T_{room} can be used to replace T.

Case 2

In this case, the temperature difference ΔT_{co} is the difference between the room temperature T_{room} and the operating temperature T_{op}. The joint is assumed to be subject to

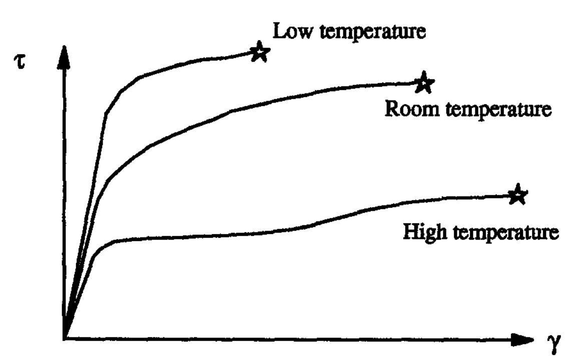

Fig. 4. Shear stress–strain behaviors at different temperatures.

tensile or compressive load P (in case of compression, it is assumed that no buckling happens). The room temperature is assumed to be the strain-free reference temperature.

For this case, the boundary conditions in eqns (12) need to be revised to include the thermal mismatch effect as follows:

$$E_o t_o\left(\frac{du_o}{dx}+\alpha_o\Delta T_{co}\right)=\frac{P}{2},\quad E_i t_i\left(\frac{du_i}{dx}+\alpha_i\Delta T_{co}\right)=0\quad \text{when } x=-0.5l \tag{38a}$$

$$E_o t_o\left(\frac{du_o}{dx}+\alpha_o\Delta T_{co}\right)=0,\quad E_i t_i\left(\frac{du_i}{dx}+\alpha_i\Delta T_{co}\right)=P\quad \text{when } x=0.5l \tag{38b}$$

or in terms of the shear strain γ

$$\frac{d\gamma}{dx}=-\frac{P}{2E_o t_o t}-\frac{(\alpha_i-\alpha_o)\Delta T_{co}}{t}\quad \text{when } x=-0.5l \tag{39a}$$

$$\frac{d\gamma}{dx}=\frac{P}{E_i t_i t}-\frac{(\alpha_i-\alpha_o)\Delta T_{co}}{t}\quad \text{when } x=0.5l. \tag{39b}$$

When γ_{final} attains its maximum value γ_{max}, the cohesive shear failure happens in the adhesive. Because γ and $d\gamma/dx$ takes its maximum value at either the $x=-0.5l$ end or the $x=0.5l$ end, thus using the modified boundary conditions (39) and eqn (35), the conditions of the adhesive cohesive failure are given by:
at the end of $x=-0.5l$

$$\left[-\frac{P}{2E_o t_o t}-\frac{(\alpha_i-\alpha_o)\Delta T_{co}}{t}\right]^2=2\lambda^2\int_0^{\gamma_{max}}\tau(\gamma,T)\,d\gamma \tag{40a}$$

or at the end of $x=0.5l$

$$\left[\frac{P}{E_i t_i t}-\frac{(\alpha_i-\alpha_o)\Delta T_{co}}{t}\right]^2=2\lambda^2\int_0^{\gamma_{max}}\tau(\gamma,T)\,d\gamma. \tag{40b}$$

Hence when one of the equations in (40) holds, the cohesive shear failure happens in the adhesive.

The bond shear strength of the joints with thermal mismatch effect is given by the lesser P from the following equations

$$P=2E_o t_o t\sqrt{2\lambda^2\int_0^{\gamma_{max}}\tau(\gamma,T)\,d\gamma}-2E_o t_o(\alpha_i-\alpha_o)\Delta T_{co} \tag{41a}$$

or

$$P=E_i t_i t\sqrt{2\lambda^2\int_0^{\gamma_{max}}\tau(\gamma,T)\,d\gamma}+E_i t_i(\alpha_i-\alpha_o)\Delta T_{co}. \tag{41b}$$

For balanced joints, e.g. $2E_o t_o=E_i t_i$ the bond shear strength takes the following simple form

$$P = 4\sqrt{E_o t_o t \int_0^{\gamma_{max}} \tau(\gamma, T)\, d\gamma} - 2E_o t_o |(\alpha_i - \alpha_o)\Delta T_{ro}|. \tag{42}$$

In eqns (41) and (42), the operating temperature is used in $\tau(\gamma, T)$. This means that the shear stress–strain curve at the operating temperature is utilized in computing the maximum strain energy density in shear for the adhesive.

The effect of thermal mismatch and the stiffness imbalance on the bond shear strength can be highlighted through introducing the following notation

$$k_{temp} = \frac{(\alpha_i - \alpha_o)\Delta T_{co}\sqrt{E_o t_o}}{2\sqrt{t \int_0^{\gamma_{max}} \tau(\gamma, T)\, d\gamma}} \tag{43}$$

and noting eqn (29), eqns (40) can be expressed as follows:

$$k_{stiffness} + k_{temp} = \pm \frac{1}{\sqrt{2}}\sqrt{1 + \left(\frac{2E_o t_o}{E_i t_i}\right)} \tag{44a}$$

or

$$k_{stiffness} - \frac{E_i t_i}{2E_o t_o} k_{temp} = \pm \frac{1}{\sqrt{2}}\left(\frac{E_i t_i}{2E_o t_o}\right)\sqrt{1 + \left(\frac{2E_o t_o}{E_i t_i}\right)}. \tag{44b}$$

In eqns (44) the stiffness of the outer adherends is prescribed as a constant while the inner adherend stiffness varies. To show the effect of the thermal mismatch and the stiffness imbalance, the following illustrative examples are discussed:

(a) for the stiffness balanced joints with $2E_o t_o = E_i t_i$, eqns (44) are replaced by

$$k_{stiffness} + k_{temp} = \pm 1, \quad \text{or} \quad k_{stiffness} - k_{temp} = \pm 1.0, \tag{45}$$

(b) for the stiffness imbalanced joints with $E_o t_o = E_i t_i$, eqns (44) are

$$k_{stiffness} + k_{temp} = \pm 1.225, \quad \text{or} \quad k_{stiffness} - 0.5k_{temp} = \pm 0.612, \tag{46}$$

(c) for the stiffness imbalanced joints with $4E_o t_o = E_i t_i$ eqns (44) are given by

$$k_{stiffness} + k_{temp} = \pm 0.866, \quad \text{or} \quad k_{stiffness} - 2k_{temp} = \pm 1.732. \tag{47}$$

Equations (45)–(47) define the failure chart for the joints as shown in Fig. 5. All the lines with negative slopes are drawn using the first equations in (45), (46) and (47), while those with positive slopes are plotted using the second equations. As the first and the second equations in (45)–(47) give the left end and the right end failure limits, respectively, the negative-sloped and positive-sloped lines are associated with the left end and the right end failure. Apparently for the stiffness balanced joints (namely $2E_o t_o = E_i t_i$), the failure chart is in a regular diamond shape, and the effect of the thermal mismatch is highlighted in terms of reducing the nondimensional bond shear strength $k_{stiffness}$. For the stiffness imbalanced joints with $E_o t_o = E_i t_i$ and $4E_o t_o = E_i t_i$, the failure charts are in twisted diamond shapes. Variation in the nondimensional thermal mismatch k_{temp} can either decrease or increase the nondimensional bond shear strength $k_{stiffness}$. For example, for $E_o t_o = E_i t_i$, $k_{stiffness}$

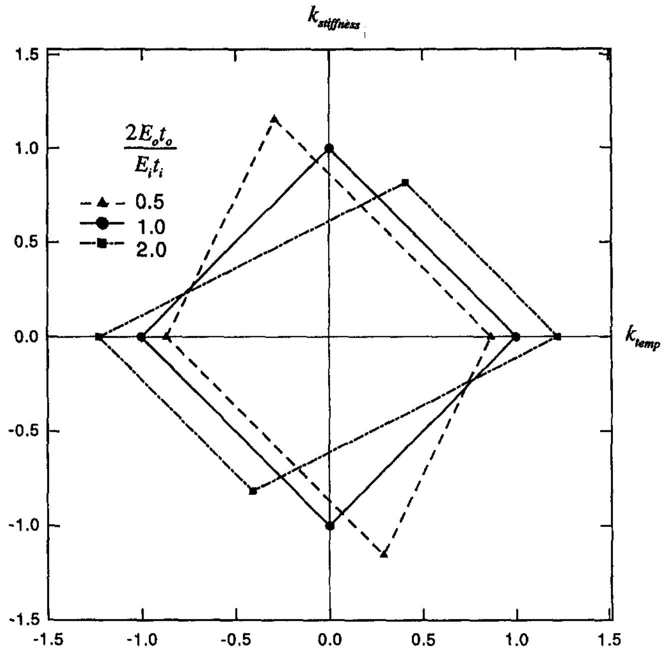

Fig. 5. Failure chart for double-lap joints with the stiffness imbalance and the thermal mismatch when prescribing the outer adherend stiffness (i.e. $2E_o t_o$ = constant).

increases when k_{temp} ranges from 0.0 to 0.408 and decreases when k_{temp} is larger than 0.408, while the failure location transfers from the left end to the right end of the overlap. This phenomena can be physically described as: when the operating temperature of the joints is close to the room temperature, $k_{stiffness}$ becomes larger as the operating temperature is elevated; when the operating temperature is sufficiently higher than the room, $k_{stiffness}$ is decreased as the operating temperature is elevated.

5. CONCLUDING REMARKS

The present investigation includes the following salient points. (a) A direct integration method is proposed for deriving a simple and efficient formula for predicting bond shear strength for adhesively bonded double-lap joints with an arbitrarily nonlinear shear stress–strain property. (b) In this formula, the bond shear strength is characterized in terms of the maximum shear strain energy density in the adhesive during the load application. (c) The thermal mismatch in the joints leads to a reduction of the bond shear strength for the balanced joints and can either decrease or increase the bond shear strength for the stiffness imbalanced joints.

REFERENCES

Baker, A. A. and Jones, R. (1988). *Bonded Repair of Aircraft Structures*. Martinus Nijhoff, Dordrecht.

Carpenter, W. C. (1991). A comparison of numerous lap joint theories for adhesively bonded joints. *J. Adhesion* **35**, 55–73.

Engineering Sciences Data Unit (1979). Inelastic shear stresses and strains in the adhesives bonding lap joints loaded in tension or shear. ESDU Item Number 79016.

Goland, M. and Reissner, E. (1944). The stresses in cemented joints. *ASME J. Appl. Mech.* **11**, A17–27.

Hart-Smith, L. J. (1973). Adhesive-bonded double lap joints. NASA CR-112235.

Hart-Smith, L. J. (1988). Design and analysis of bonded repairs for metal aircraft structures. In *Bonded Repair of Aircraft Structures* (Edited by A. A. Baker and R. Jones) Chapter 3. Martinus Nijhoff, Dordrecht.

Jones, R., Chiu, W. K. and Paul, J. (1993). Designing for damage tolerant bonded joints. *Composite Structures* **25**, 201–207.

Krieger, R. B. Jr (1988). Stress analysis concepts for adhesive bonding of aircraft primary structures. In *Adhesively Bonded Joints: Testing, Analysis and Design*, ASTM STP981 (Edited by W. S. Johnson), pp. 264–275. ASTM, Philadelphia.

Thrall, E. W. (1979). Primary adhesively bonded structure technology (PABST): Design handbook for adhesive bonding. USAF Technical Report, AFFDL-TR-79-3119.

Volkersen, O. (1938). Die Nietkraftverteilung in zugbeanspruchten Nietverbindungen mit konstanten Laschenguerschnitten. *Luftfahrtforschung* **15**, 41–47.